CALCULUS (Assignment 1)
- Unit 1 & 2

TBS → 26th Nov.

① $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$\lim_{h \to 0} \frac{2(9 - 6h + h^2) - 18}{h}$

$\lim_{h \to 0} \frac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$

$\lim_{h \to 0} \frac{2\cancel{h}(-6 + h)}{\cancel{h}}$

$\lim_{h \to 0} -12 + 2h$

Ans) -12

② $g(x) = \begin{cases} 2x & , x < 6 \\ x - 1 & , x \geq 6 \end{cases}$

at $x = 4$, $x = 6$.

$g(x)_{x \to 4^-} = 2x.$

$= 8$

$\lim_{x \to 4^+} = x - 1$

$= 6 - 1$

$= 5$

g(x) is discontinuous.

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(3) $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$\lim_{x \to 9} \dfrac{(x-9)}{3-\sqrt{x}} \dfrac{(3+\sqrt{x})}{(3+\sqrt{x})}$

$\lim_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{9-x}$

$\cancel{\lim_{x \to 9} \dfrac{3x + x\sqrt{x} - 27 + 9\sqrt{x}}{9-x}}$

$\lim_{x \to 9} \dfrac{-\cancel{(9-x)}(3+\sqrt{x})}{\cancel{9-x}}$

$\Rightarrow -(3+\sqrt{9})$

$\Rightarrow -(3+3)$

Ans $\Rightarrow -6$

(4) $f(x) = \dfrac{4x+5}{9-3x}$ at $x = -1, 0, 3$ (LAST)

(6) $g(y) = \begin{cases} y^2+5 & , y < -2 \\ 1-3y & , y \geq -2 \end{cases}$

a) $\lim_{y \to 6} g(y) \quad 1-3y$

$1-3(6)$

Ans -17.

b) $\lim_{y \to -2} g(y) \quad y^2+5$

Ans 9

(7) $r(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$$\frac{d}{dx}\left\{(4t^2 - t)(t^3 - 8t^2 + 12)\right\}$$

using product rule,

$$(4t^2 - t)\frac{d}{dx}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dx}(4t^2 - t)$$

$$(4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$$

$-67t^3$
$-65t^3$
$-132t^3$

$$\Rightarrow 20t^4 - 132t^3 + 24t^2 + 96t - 12.$$

Ans) $20t^4 - 132t^3 + 24t^2 + 96t - 12$

(8) $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$$\frac{2w^2 + 1\,\frac{d}{dx}(3w + w^4) - (3w + w^4)\frac{d}{dx}(2w^2 + 1)}{(2w^2 + 1)^2}$$

$$\Rightarrow \frac{(2w^2 + 1)(4w^3 + 3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

$$\Rightarrow \frac{8w^5 + 6w^2 + 4w^3 + 3 - [12w^2 + 4w^5]}{(2w^2 + 1)^2}$$

Ans $\Rightarrow \dfrac{12w^5 - 6w^2 + 4w^3 + 3}{(2w^2 + 1)^2}$

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(9) $f(x) \; 2e^{x} - 8^{x}$

$f'(x) = \frac{d}{dx}(2e^{x} \; 8^{x})$

$\Rightarrow -2e^{x} - 8^{x} \log_e 8$

Ans) $\cancel{(2} -(2e^{x} + 8^{x} \log_e 8)$

(10) $y = z^5 - e^{z} \ln(z)$ (LAST)

(11) a) $f(x) = (6x^2 + 7x)^4$

$f'(x) = 4(6x^2 + 7x)^3 \times (12x + 7)$

Ans) $4(6x^2 + 7x)^3 (12x + 7)$

b) $f(t) = 5 + e^{4t + t^7}$

$f'(t) = 0 + e^{4t + t^7} \times (4 + 7t^6)$

Ans $= (4 + 7t^6) e^{4t + t^7}$

(12) $z = \ln(7 - x^3)$

$\frac{dz}{dx} = \frac{d}{dx} \log(7 - x^3)$

$= \frac{1}{(7 - x^3)} \times (-3x^2)$

$\frac{d''z}{dx''} = \cancel{-6x} \; \frac{d}{dx} \frac{(-3x^2)}{(7 - x^3)}$

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$$\frac{d}{dx}\,\frac{(-3x^2)}{7-x^3}$$

$$\Rightarrow \frac{(7-x^3)(-6x)-(-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$\Rightarrow \frac{-42x+6x^4-(9x^4)}{(7-x^3)^2}$$

Ans $\Rightarrow \dfrac{-42x+6x^4-9x^4}{(7-x^3)^2}$

b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$Q'(v) \Rightarrow \dfrac{-2[4(6+2v-v^2)^3[2-2v]]}{(6+2v-v^2)^8}$

$$\Rightarrow \frac{-8[6+2v-v^2]^3[2-2v]}{(6+2v-v^2)^8}$$

$$\Rightarrow -8[6+2v-v^2]^{-5}[2-2v]$$

$$\Rightarrow [-16+16v][6+2v-v^2]^{-5}$$

$Q''(v) \Rightarrow [6+2v-v^2]^{-5}[16] + [16v-16][-5(6+2v-v^2)^{-6} \times (2-2v)]$

Ans) $16(6+2v-v^2)^{-5} + (2-2v)(16v-16)-5(6+2v-v^2)^{-6}$

c) $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2}(2t)$

$f''(t) = \frac{(1+t^2)2 - 2t(2t)}{(1+t^2)^2}$

$\Rightarrow \frac{2+2t^2-4t^2}{(1+t^2)^2}$

$\Rightarrow \frac{2-2t^2}{(1+t^2)^2}$

Ans $\Rightarrow \frac{2(1-t^2)}{(1+t^2)^2}$

Q13. $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$
$= 3(x^2 - 2x - 3)$
$= 3(x^2 - 3x + x - 3)$
$= 3(x(x-3) + 1(x-3))$
$= 3(x-3)(x+1)$

$f'(x) = 0$ and
$x = 3$ ~~or~~ -1

$\therefore x = 3$ and $x = -1$ could be pts of local maxima or minima.

$f''(x) = 6x - 6$
$x = 3$, $12 > 0$
$\therefore x = 3$ pt of local minimum

$f''(x) = 6x - 6$
$x = -1 \Rightarrow -12 < 0$
$\therefore x = -1$ local maxima.

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(14) $4x^3 - 18x^2 + 24x - 7$

$f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x^2 - 2x - x + 2)$

$= 12(x(x-2) - 1(x-2))$

$= 12(x-1)(x-2)$

$f'(x) = 0$

$x = 1$ and $x = 2$ are the points of local minima or maxima.

$f''(x) = 24x - 36$

$x = 1, \ -12 < 0$

$\therefore x = 1$ is pt of local maxima.

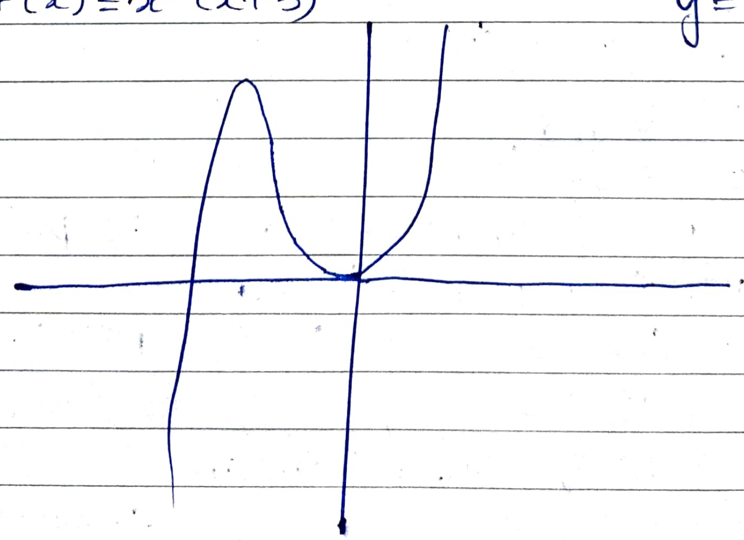
$f''(x) = 24x - 36$

$x = 2, = 48 - 36$

$12 > 0$

$\therefore x = 2$ is pt of local minima.

(15) $f(x) = x^2(x+3)$ $\qquad$ $y = x^2(x+3)$



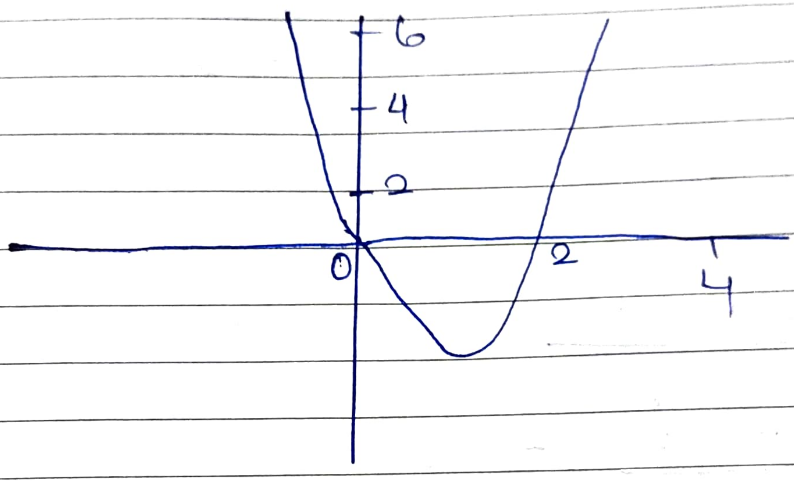
(10) $y = z^5 \cdot e^z (\ln z)$

$\log y = 5\log z - 7\log e - \log(\log(z))$

$\frac{1}{y}\frac{dy}{dx} = \frac{5}{z} - 1 - \frac{1}{\log(z)} \times \frac{1}{z}$

$\frac{dy}{dx} = z^5 e^z \ln(z) \left[\frac{5\log z - z\log z - 1}{7\log(z)}\right]$

(16) $f(x) = 3x^2 - 6x$



(4) $f(x) = \frac{4x+5}{4-3x}$

i) $x = -1$

$\lim_{x \to -1} \frac{4(-1)+5}{9-3x} = \frac{1}{12}$

$f(-1) = \frac{1}{12}$

$\lim_{x \to -1} f(x) = f(-1)$

continuous at $x = -1$

ii) $x = 0$

$\lim_{x \to 0} \frac{4(0)+5}{9-(0)3} = \frac{5}{9}$

$\lim_{x \to 0} f(x) = f(0)$

it is continuous $x = 0$

$x = 3$

$\lim_{x \to 3^-} \frac{4x+5}{9-3x}$

$\lim_{h \to 0} \frac{4(3-h)+5}{9-3(3-h)}$

$\lim_{h \to 0} \frac{17-4h}{9-9+3h}$

$\lim_{h \to 0} \frac{17-4h}{3h}$

$LHL = \lim_{h \to 0} \frac{17}{h} - \lim_{h \to 0^-} \frac{4h}{h} = \lim_{h \to 0} \frac{17}{h} - 4$

$= \infty - 4$

$= \infty$

$\lim_{x \to 3^+} \frac{4x+5}{9-3x}$

$\lim_{h \to 0} \frac{12+5+4h}{-h}$

$= \lim_{h \to 0} \frac{17+4h}{-h} = -\infty$

$x \in R - \{3\}$

$\therefore LHL \neq RHL$

$\therefore f(x)$ is discontinuous at $x = 3$.

(5) $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

(taking common e^{-2x} in both num & denominator

$\lim_{x \to \infty} \dfrac{6e^{6x} - 1}{8e^{6x} - e^{4x} + 3e^{-x}}$

$\cancel{\lim_{x \to \infty} \dfrac{6e^{6x} - 1}{}}$

$\lim_{x \to \infty} \dfrac{\cancel{e^{6x}}\left(6 - 1/e^{6x}\right)}{\cancel{e^{6x}}\left(8 - \dfrac{1}{e^{2x}} + \dfrac{3}{e^{5x}}\right)}$

$\lim_{x \to \infty} \dfrac{6 - 1/e^{6x}}{8 - 1/e^{2x} + 3/e^{5x}}$

$\dfrac{1}{e^{6x}} = \dfrac{1}{e^{2x}} = \dfrac{1}{e^{5x}} = 0$, when $x \to \infty$

$\lim_{x \to \infty} \dfrac{6}{8} = \dfrac{3}{4}$